

Synthetic Division

• Shortcut for polynomial long division!
(only works for binomials!) ex $(x-2)$
 $(x+3)$

EX) $(2x^2 + 7x + 9) \div (x+2)$

dividend \downarrow divisor \swarrow

$$\begin{array}{r|rrr} -2 & 2 & 7 & 9 \\ & \downarrow + & -4 & -6 \\ \hline & 2 & 3 & 3 \end{array}$$

(-2×2)

\uparrow \uparrow \uparrow
x constant Remainder
always the last #

STEPS!!

- ① Write coefficients of the dividend (including zero placeholder if needed)
- ② Use opposite sign of the number in the divisor.
- ③ Bring down the first coefficient!
- ④ Multiply the divisor with the coefficient. Place it above the line, in the next column
- ⑤ Add!
- ⑥ Repeat!

For final answer!
Step down 1 degree
and continue on!

$$\text{Solution: } 2x + 3 + \frac{3}{x+2}$$

$$x) (3x^4 - x^3 + 5x - 1) \div (x - 3)$$

+0x² ← add missing term

3		^{x⁴} 3	-	^{x³} 1	+	^{x²} 0	+	^{x¹} 5	-	^{x⁰} 1
		↓		+	9	+	24	72		231
		3		8		24		77		230
		^{x³}		^{x²}		^x		constant		↑ Remainder

Solution: $3x^3 + 8x^2 + 24x + 77 + \frac{230}{x-3}$

Ex) $(3x^2 + 9x - 2) \div (3x - 1)$

Hmm... Leading coefficient is not 1?

Rewrite: $(3x^2 + 9x - 2) \div (x - 1/3)$

We need it to be 1.

1/3		3	+	9	-	2
		↓		1		3.333
		3		10		1.33
		^x		constant		↑ remainder

$$3x - 1 = 0$$

$$+1 \quad +1$$

$$\frac{3x}{3} = \frac{1}{3}$$

$$x = 1/3$$

Solution: $3x + 10 + \frac{1.33}{(x - 1/3)}$

Factor Theorem:

- A binomial is a factor of a polynomial if we get a zero remainder.

$$(3x^3 + 2x^2 - 33) \div (x-2)$$

✓
+0x

2	3	2	0	-33
	↓	6	16	32
	3	8	16	-1

$(x-2)$ is not a factor, because remainder is not zero

Better yet!! There is a short cut!!

Remainder Theorem: Plug the opposite value of the divisor into the polynomial to find the remainder.

$$\text{ex) } (3x^3 + 2x^2 - 33) \div (x-2)$$

$$P(2) = 3(2)^3 + 2(2)^2 - 33 = -1$$

Everyplace you see x , plug in 2!

just like above, remainder matches.

1) $(2x^3 - 3x^2 - 8x - 3)$ is $(2x + 1)$
a factor?

$$P(-.5) = 2(-.5)^3 - 3(-.5)^2 - 8(-.5) - 3$$

$$P(-.5) = 0$$

↑ 0 is my remainder!

Meaning $(2x + 1)$ is a factor of
 $(2x^3 - 3x^2 - 8x - 3)$!

$$2x + 1 = 0$$

$$-1 \quad -1$$

$$\frac{2x}{2} = \frac{-1}{2}$$

$$x = -\frac{1}{2}$$