Solving Rational Equations

What are Rational Equations?
Equations with fractions & variables in the denominator

Proportions: one fraction is set equal to another.

(Solving Method:) Cross Multiplication

Ex) $\frac{20}{3x-5} = \frac{5}{x-2}$

$5(3x-5) = 20(x-2)$

$15x - 25 = 20x - 40$

$-15x \quad -15x$

$-25 = 5x - 40$

$+40 \quad +40$

$\frac{15}{5} = \frac{5x}{5}$

$x = 3$

Check to see if extraneous!

$\frac{20}{3(3)-5} = \frac{5}{3-2}$

$5 \frac{20}{4} = \frac{5}{1} 5$ ✓

ex) $\frac{x}{x^2-2} = \frac{-1}{x}$

$-1(x^2-2)$

$x^2 = 2 - x^2$

$+x^2 \quad +x^2$

$\frac{2x^2}{2} = \frac{2}{2}$

$\sqrt{x^2} = \sqrt{1}$

$x = 1, -1$

Check 1:

$\frac{1}{-1(1)^2-2} = \frac{-1}{1}$ ✓

Check -1:

$\frac{-1}{(-1)^2-2} = \frac{-1}{-1}$ ✓

-1

You try! Solving Rational Equations

Solve the following by cross-multiplication. Be sure to check for extraneous solutions!

1. $\frac{x}{9} = \frac{7}{3}$

$3x = 63$

$x = 21$ ✓

2. $\frac{x-5}{15} = \frac{4}{5}$

$60 = 5x - 25$

$85 = 5x$

$x = 17$ ✓

3. $\frac{3x+1}{x-1} = \frac{5x-5}{x-1}$

$3x^2 - 2x - 1 = 5x^2 - 10x + 5$

$0 = 2x^2 + 8x + 6$

$0 = 2x^2 + 2x + 6x + 6$

$0 = 2x(x+1) + 6(x+1)$

$0 = (2x+6)(x+1)$

$x = 3$ ✓, -1 ✗

OR

$3x + 1 = 5x - 5$

$6 = 2x$

$x = 3$ ✓

Since same base!!

4. $\frac{12}{x} = \frac{24}{x+5}$

$12x + 60 = 24x$

$60 = 12x$

$x = 5$ ✓

5. $\frac{3}{5-3x} = \frac{1}{2}$

$6 = 5 - 3x$

$1 = -3x$

$x = -1/3$ ✓

6. $\frac{2}{x-3} = \frac{1}{x^2-2x-3}$

$x - 3 = 2x^2 - 4x - 6$

$0 = 2x^2 - 5x - 3$

$0 = 2x^2 - 6x + 1x - 3$

$2x(x-3) + 1(x-3)$

$x = -1/2$ $x = 3$ ✗

7. $\frac{x+3}{x+1} = \frac{15}{x+7}$

$(x+3)(x+7) = 15x + 15$

$x^2 + 10x + 21 = 15x + 15$

$x^2 - 5x + 6 = 0$

$(x-3)(x-2) = 0$

$x = 3, 2$ ✓ ✓

8. $\frac{x-1}{7} = \frac{2x-2}{3x-1}$

$14x - 14 = 3x^2 - 4x + 1$

$0 = 3x^2 - 18x + 15$

$0 = 3x^2 - 3x - 15x + 15$

$0 = 3x(x-1) - 15(x-1)$

$0 = (x-1)(3x-15)$

$x = 1, 5$ ✓