Unit 4 Review - Polynomials NAME Key

Polynomial Division

Divide using either long division or synthetic division (when possible).

1. $(9x^3 - 2x^2 + 5x + 4) \div (x - 3)$

3 | 9 -2 5 4

$9x^2 + 25x + 80 + \frac{244}{x-3}$

2. $(6x^3 + 19x^2 + 7x - 12) \div (2x + 3)$.

$3x^2 + 5x - 4$

3. $(12x^3 - 7x^2 - 38x + 35) \div (4x - 5)$

$3x^2 + 2x - 7$

4. $(x^4 + 7x^3 - 6x + 2) \div (x + 4)$

$x^3 + 3x^2 - 12x + 42 - \frac{166}{x+4}$

Remainder/Factor Theorem

Determine which are factors of $2x^{91} - x^{90} - 10x^{89}$.

5. $3x + 1$

NO

6. $2x - 5$

YES

7. $x + 2$

YES

Polynomial Vocabulary

Classify each polynomial by the degree and by the number of terms.

8. $7x^3 - 2x$

Cubic
Binomial

9. $-10x^4 - 3x^3 + 2$

Quartic
Trinomial

10. 7

Constant
Monomial

Solve Polynomials

Determine all real and complex solutions.

15. $x^3 - 5x^2 + 3x - 15 = 0$

$x = 5, i\sqrt{3}, -i\sqrt{3}$

16. $x^4 - 3x^3 - 24x^2 + 80x = 0$

$x = 0, 4, -5$

17. $x^3 + 64 = 0$

$x = -4, 2 + 2i\sqrt{3}, 2 - 2i\sqrt{3}$

18. $x^3 + 5x^2 + 10x + 24 = 0$

$x = -4, \frac{-1 + i\sqrt{23}}{2}, \frac{-1 - i\sqrt{23}}{2}$

Applications

19. The weight of an ideal round-cut diamond can be modeled by $w = 0.0074d^3 - 0.087d^2 + 0.32d$, where w is the diamond's weight (in carats) and d is its diameter (in millimeters). According to the model, what is the weight of a diamond with a diameter of 12 millimeters?

4.0992

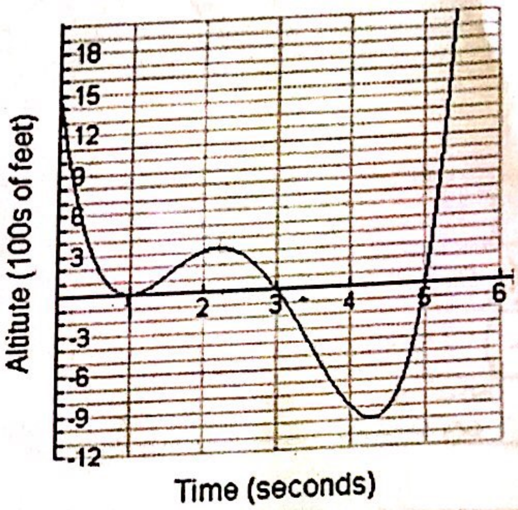
20. The profit P (in millions of dollars) for a t-shirt manufacturer can be modeled by $P = -x^3 + 5x^2 + 9x$, where x is the number of t-shirts produced (in millions). Currently, the company produces 5 million t-shirts and makes a profit of $45,000,000. What lesser number of t-shirts could the company produce and still make the same profit?

3,000,000

21. A box has a height of $x-4$ inches and a length of $x+3$ inches. If the volume of the box is $2x^3-3x^2-23x+12$ cubic inches, determine the width of the box.

$2x-1$

22. When fighter pilots train for dog-fighting, a "hard-deck" is usually established below which no competitive activity can take place. The polynomial graph given shows Maverick's altitude (y in 100s of feet) above and below this hard-deck during a 5 second (x) interval.



a. What is the lowest possible degree of this polynomial?

4

b. How many total seconds was Maverick above the hard-deck during the first 5 seconds?

3 seconds

c. After how many seconds is Maverick 300 feet above the hard-deck?

2.25seconds

d. Determine the equation of the function in factored form.

$(x-1)^2(x-3)(x-5)$

Rates of Change

23. Find the average rate of change from ***x* = – *1*** to ***x* = *3*** for each of the functions below.

a. $a(x)=2x+3$

8

b. $b(x)=x^2-2$

8

c. $c(x)=2^x-1$

7.5

d. Which function has the greatest average rate of change over the interval **[– 1, 3]**?

$2x+3$ & x^2-2

24. In general as $x \to \infty$, which function eventually grows at the fastest rate?

a. $a(x)=3x$

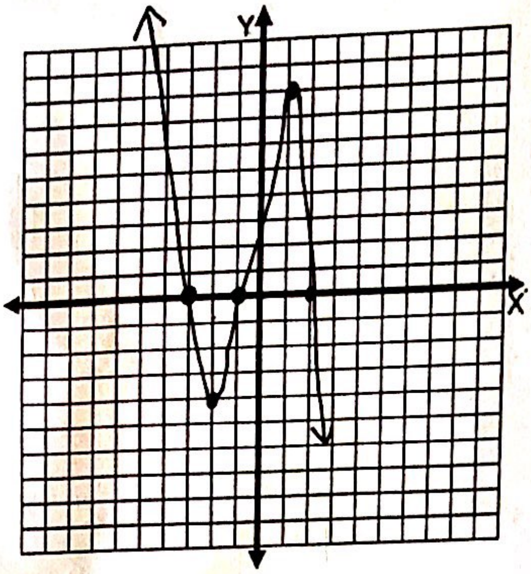
b. $b(x)=x^3$

c. $c(x)=3^x$

Zeroes and Multiplicity, Extrema, Intervals for Increasing/Decreasing/Positive/Negative

For each graph and equation, determine all key features.

11.



Zeroes: x = -3, -1, 2

Extrema: Rel Min (-2, -4) Rel Max (1, 8)

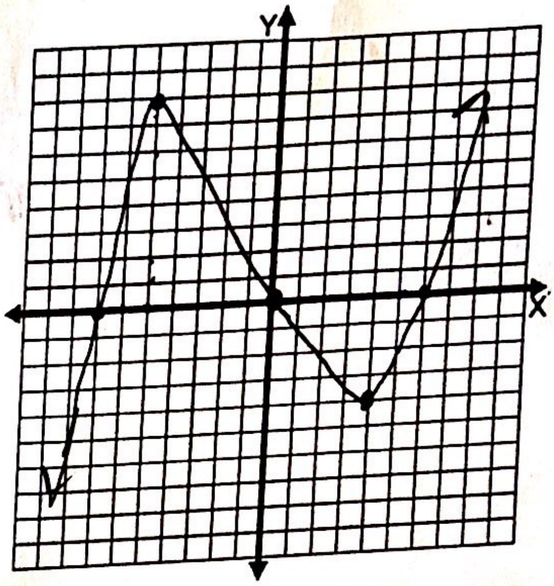
Pos/Neg: Pos (-∞, -3) & (-1, 1) Neg (-3, -1) & (1, ∞)

Inc/Dec: Inc (-2, 1) Dec: (-∞, -2) & (1, ∞)

End Behavior: As x → -∞ y → ∞ As x → ∞ y → -∞

Degree: 3

12.



Zeroes: x = -7, 0, 6

Extrema: Rel Max (-5, 7) Rel Min (4, -4)

Pos/Neg: Neg (-∞, -7) & (0, 6) Pos: (-7, 0) & (0, ∞)

Inc/Dec: Inc: (-∞, 5) & (4, ∞) Decre: (-5, 4)

End Behavior: As x → -∞ y → -∞ As x → ∞ y → ∞

Degree: 3

13. $y = -2(x+1)^2(3x-1)$

Zeroes: x = -1, 1/3

Extrema: Rel Min (-1, 0), Rel Max (-.11, 2.11)

Pos/Neg: Positive: (-∞, 1/3) Neg (1/3, ∞)

Inc/Dec: Decrease: (-∞, -1) & (0, ∞) Increase: (-1, 0)

End Behavior: As x → -∞ y → ∞ As x → ∞ y → -∞

Degree:

3

14. $y = x^3(x-2)(x-3)$

Zeroes: x = 0, 2, 3

Extrema: Rel Max (1.67, 2.06) Rel Min (2.6, -4.2)

Pos/Neg: Pos: (0, 2) & (3, ∞) Neg (-∞, 0) (2, 3)

Inc/Dec: Incr: (-∞, 1.4) & (2.6, ∞) Decrease (1.4, 2.6)

End Behavior: As x → -∞ y → -∞ As x → ∞ y → ∞

Degree: 5