Final Exam Review Sheet

Unit 1: Functions and Inverses

3 Methods for Solving Systems:

1. Substitution:
   - Solve one equation for a variable. (Y= or X=)
   - Substitute that equation into your other equation and solve.
   - Plug your answer into either equation to solve for the remaining variable.
2. Elimination:
   - Make sure the equations are lined up with common variables and constants.
   - If needed multiply an entire equation so that x or y has the same coefficient with opposite signs.
   - Add straight down and solve.
   - Plug answer into either equation and solve for the remaining variable.
3. Graphing:
   - Isolate for y in each equation.
   - Plug into Y=
   - Where do the equations intersect?
   - Calculator Steps (2nd → Trace → 5) Go to intersection and hit enter 2 times.

Three Types of Solutions:

- Ordered Pair(s) - Where do the functions intersect?
- Infinitely Many - You have the same equation. Algebraically you would get a true statement. (Example: 0=0, 4=4, 2=2)
- No Solution- The equations never intersect. Algebraically you would get a false statement. (Example: 3=6, -1= 5, 0=9)

Systems of Inequalities: We find the solutions to our system when our shading overlaps.

Lines:

- > or < Dotted Line
- ≥ or ≤ Solid Line

Shading:

- Y> or Y ≥ Shade Above
- Y< or Y ≤ Shade Below

Absolute Value Functions:

- Absolute Value Bars in your Calculator (2nd → 0 → Enter)

1. Isolate the absolute value
2. Drop the absolute value symbol and set up two equations
   - One positive, one negative
3. Solve both equations

Inverses: Equations that undo one another and therefore “swap” domain and range. X and Y “flip” or change positions.

1. Change “f(x)” notation to “y”
2. Switch the x and y variables in the function.
3. Solve the equation for y.
4. Replace y with $f^{-1}(x)$

Function Operations:

The value in parenthesis is what we are plugging into the equation. f(2) plug 2 into the equation.

- Addition: $(f + g) = f(x) + g(x)$
- Subtraction: $(f - g) = f(x) - g(x)$

- Multiplication: $(f \cdot g) = f(x) \cdot g(x)$
- Compositions: $(f \circ g) = f(g(x))$

Unit 2: Exponential and Logarithmic Functions

Properties of Logs:

- Move from Log to Exponential Form and vise versa to help us solve for variables
    - If $y = b^x$ then $log_b y = x$
- Logs have base 10
- Natural logs have base e (2.718)
- $log_b mn = log_b m + log_b n$
- $log_b \frac{m}{n} = log_b m - log_b n$
- $log_b m^p = p log_b m$

Solving Exponential Equations:

1. Isolate the exponential term
2. Take the natural log of both sides
3. Use the exponent property to move it in front of the natural log
4. Solve for the variable!

Graphing Logarithmic and Exponential Functions:

- Asymptote: A line that we approach, but never cross. Boundary line.
    - Exponential: Horizontal Asymptote being added/ subtracted at end of equation
    - Logarithm: Vertical Asymptote is the number inside with x, set =0 then solve.

Exponential Functions:

- $a = pb^t$
- p = initial amount
- b = growth/decay factor
    - Growth → adding 1 (b >1)
    - Decary → subtracting from 1 (0 < b < 1)
- t = time period

Compound Interest:

- Compounded Over Time Period: $A = P(1 + \frac{r}{n})^{nt}$
- Compounded Continuously: $A = Pe^{rt}$
- Variables:
    - A: Final amount
    - P: Principle (initial amount)
    - r: interest rate → as a decimal
    - n: number of times compounded in on year
    - t: time in years
- Compounding and n values:
    - n = 1 Compounded Yearly
    - n = 2 Compounded Semi Annually
    - n = 4 Compounded Quarterly
    - n = 12 Compounded Monthly

Unit 3: Modeling with Geometry

Nets: Flattened 3D shape

Cross Sections: the surface that is created by cutting a 3D solid with a 2D plane.

Surface Area: If we were to cover a 3D shape in wrapping paper. (The sum of all side areas.)

- Helpful Area Formulas
    - Circle: $A = \pi r^2$
    - Triangle: $A = \frac{1}{2}bh$
    - Rectangle: $A = bh$
    - Trapezoid: $A = \frac{1}{2}(b_1 + b_2)h$
    - SA of a Sphere $= 4\pi r^2$
    - SA of a Cone $= \pi r^2 + \pi rl$ → where l is the slant height

Volume: If we were to fill a 3D shape with sand.

- Prisms and Cylinders: V= Area of the base* Height
- Pyramids and Cones: V= $\frac{1}{3}$ Area of the base* Height
- Volume of a Sphere = $\frac{4}{3}\pi r^3$

Unit 4: Polynomials:

Long Division:

1. Divide 2. Multiply 3. Subtract 4. Bring Down 5. Repeat!

- Before you begin, make sure you are in standard form (highest power → lowest power)
- Make sure missing terms are written with a zero coefficient.

Synthetic Division:

1. Write coefficients of the dividend (what you are dividing) [including zero placeholder if needed.]
2. Use opposite sign of number is the divisor (what you are dividing by)
3. Bring down the first coefficient.
4. Multiply the divisor with the coefficient, place it above the line in the next column.
5. Add; Repeat.
6. Last number is your remainder, as you move to the left step up a degree for each term.

Remainder Theorem: Plug the opposite value of the divisor into the polynomial to find the remainder. If it is zero, it is a factor. If it is not zeros, then the remainder is the number remaining over the divisor!

Key Features/Vocabulary:

- Zero: Referred to as “root” or “Solution” or “x-intercept.”
- Term: Single number or variable
    - 1 Term: Monomial
    - 2 Terms: Binomial
    - 3 Terms: Trinomial
    - 4 Terms: Polynomial
- Degree: Highest exponent of a polynomial
    - Degree 0: Constant
    - Degree 1: Linear
    - Degree 2: Quadratic
    - Degree 3: Cubic
    - Degree 4: Quartic
    - Degree >4: Polynomial
- Multiplicity: How often a zero occurs. Can tell my factored form with exponent or looking at graph.
    - Multiplicity 1: Single Root → Crosses through x-axis
    - Multiplicity 2: Double Root → Bounced off x-axis
    - Multiplicity 3: Triple Root --? Crosses x-axis, but flattens/sways
    - Even → Bounced Odd→ Crosses

- Extrema: High and low point on a graph
    - Absolute Extrema: the highest or lowest points on a function
    - Relative Extrema: points higher or lower than nearby points
- Increasing/Decreasing: looking at x-values. Look at slope!
- Positive/Negative: looking at y-values. Are we above or below x-axis?

End Behavior: What the graph does at each end. You can look at leading coefficient and degree.

- Positive/Even (As $x \to -\infty$, $y \to \infty$, As $x \to \infty$, $y \to \infty$)
- Negative/Even (As $x \to -\infty$, $y \to -\infty$, As $x \to -$, $y \to -\infty$)
- Positive/Odd (As $x \to -\infty$, $y \to -\infty$, As $x \to \infty$, $y \to \infty$)
- Negative/Odd(As $x \to -\infty$, $y \to \infty$, As $x \to \infty$, $y \to -\infty$)
- Even points in same direction; Odd points in different directions

Solving Polynomials:

- Graph polynomial. Look for any visible zeros (2nd → Trace → 2. Left → Right → Enter)
- Factor or use Quadratic Formula if needed. $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

Unit 5: Reasoning with Geometry

Geometric Properties:

- Reflexive Properties: Share the same side, or angle.
- Vertical Angles: opposite angles formed by intersecting lines (Congruent)
- Linear Pair: adjacent angles that form a line (Supplementary)
- Relationships formed by parallel lines and transversals
    - Corresponding Angles: Same side, same position (Congruent)
    - Alternate Interior Angles: opposite side of transversal, inside parallel lines (Congruent)
    - Alternate Exterior Angles: opposite side of transversal, outside parallel lines (Congruent)
    - Consecutive Interior Angles: same side of transversal, inside parallel lines (Supplementary)
    - Consecutive Exterior Angles: same side of transversal, outside parallel lines (Supplementary)

Triangle Congruence Postulates:

- Angle-Side-Angle
- Side-Angle-Side
- Side-Side-Side
- Angle-Angle-Side
- Hypotenuse-Leg
- No AAA (roadside assistance) or SSA (Donkeys!)

Properties of Parallelograms:

- Opposite sides are parallel
- Opposite sides are congruent
- Consecutive angles are supplementary
- If one angle is right, all angles are right
- Diagonals bisect each other
- Diagonals create two congruent triangles

Properties of Quadrilaterals:

- Rectangles
    - All properties of parallelograms
    - 4 right angles
    - Diagonals are congruent

- Rhombus
    - All properties of parallelograms
    - Diagonals bisect angles
    - Diagonals are perpendicular
- Square:
    - All properties of parallelograms
    - All properties of a rectangle
    - All properties of a rhombus
- Isosceles Trapezoid:
    - One set of parallel sides
    - Base angles are congruent
    - Diagonals are congruent
    - Opposite angles are supplementary
- Trapezoid Midsegment: Midsegment is half the sum of the lengths of the bases.
- Kite:
    - Diagonals are perpendicular
    - Diagonals bisect opposite angles
    - One pair of opposite angles are congruent
    - One diagonal bisects the other.

Unit 6: Circles

Angles:

- Inscribed Angles: Angle formed by two chords, half the arc measure
- Central Angles: Angles formed by two radii, equal to arc measure

Equation of a Circle:

- $(x-h)^2+(y-k)^2=r^2$
- Center: (h,k)
- Radius: r

Arc Length and Area of a Sector:

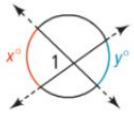
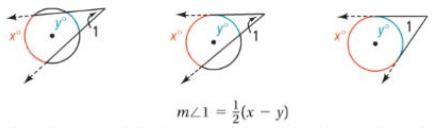
Arc Length: $s=\frac{\theta\pi r}{180}$, θ is the angle measure

Area of a Sector: $A=\frac{\theta\pi r^2}{360}$

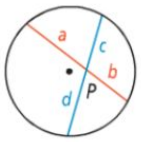
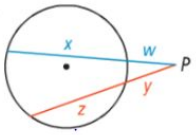
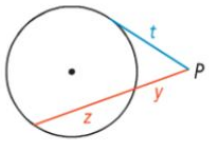
Tangents:

- Tangent line intersects a circle at exactly one point.
- When a tangent line intersects with a radius, it forms a right angle
- When two tangent lines share a common endpoint outside the circle, the segments are congruent.

Angles Theorems:

Theorem 1:	Theorem 2:
The measure of an angle formed by two lines that intersect inside a circle is half the sum of the measures of the intercepted arcs.  $m\angle 1=\frac{1}{2}(x+y)$	The measure of an angle formed by two lines that intersect outside a circle is half the difference of the measures of the intercepted arcs.  $m\angle 1=\frac{1}{2}(x-y)$

Segment Theorems:

Theorem 1:		
For a given point and circle, the product of the lengths of the two segments from the point to the circle is constant along any line through the point and the circle.		
		
$a \cdot b = c \cdot d$	$(w + x)w = (y + z)y$	$(y + z)y = t^2$

Unit 7: Rationals

Simplifying Rationals:

- Factor the numerator and denominator completely.
- Cross out any terms that are in both the numerator and denominator.

Multiplying Rationals:

- Factor the numerator and denominator completely.
- “Smush” the two rationals together to form one large fraction. Simplify!

Dividing Rationals:

- KEEP-CHANGE-FLIP! Keep the original rational, change from division to multiplication, flip the last rational!
- Multiply! Simplify!

Adding/Subtracting Rationals:

- Make common denominators!
    - If denominators are not common, factor the denominator completely. Find which factors each rational is missing.
    - Multiply each rational by that factor in both the numerator and the denominator.
    - Add or subtract across the top, keep denominators the same!
    - Be careful with subtracting. Make sure you are subtracting each term! (Distribute your negative!)

Solving Rationals:

- Simplify so that we have one rationa on each side of the equal sign.
- Cross Multiply! Solve for you variable!
- Don’t forget to check your solutions! If it does not satisfy the statement we call this extraneous!

Graphing Key Features:

- Holes: Factors that cross out when simplifying. (Set equal to zero and solve!)
- Vertical Asymptotes: Factors remaining in the denominator after simplifying! (Set equal to zero and solve!)
- Horizontal Asymptotes: BOBO BOTN EATS DC
    - BOBO: Degree is bigger in the denominator (bottom) Y = 0
    - BOTN: Degree is bigger in the numerator (top) NONE, no HA.
    - EATS DC: Degrees are equal, divide coefficients.

Unit 8: Trigonometry

- Angles
    - Sketching in Standard Position:
        - Start along x-axis axis: Mark terminal side where the angle ends.

        - Draw path of the angle. Is it positive? Move up! Negative? Move Down! If it is greater than 360, make sure you include the number of rotations (using your spirals!)
    - Converting:
        - Angles to Radians? Divide by 180, Slap π on top, in numerator!
        - Radians to Degrees? Plug 180 in place of π, simplify!
- Right Triangle Trig
    - SOH CAH TOA!
    - Make sure you are in Degree Mode
    - Solving for angles? Need Inverse (2nd→ Trig Function)
    - Reciprocals:
        - Sine (SIN) → Cosecant (CSC)
        - Cosine (COS) → Secant (SEC)
        - Tangent (TAN) → Cotangent (COT)
- Looking at our Unit Circle:
    - (X, Y) → (Cosine, Sine) Think Alphabetical order!
    - Tangent → $\frac{Sine}{Cosine}$
- Tips and Tricks to filling out Unit Circle
    - Start with 4 main points along axes.
    - Filling in Angles: Big Gap (+30); Small Gap (+15)
    - Convert to Radians
    - Left Hand Trick: Put the numbers in the following form: $\frac{\sqrt{\#}}{2}$
    - Color code: Think about quadrants to get correct signs!
- Graphing Key Features:
    - Sine: Starts at midline. If it moves up, it is positive. If it goes down, it is negative.
    - Cosine: Starts above or below the midline. If it is above the midline, then it is positive, if it is below the midline, it is negative.
    - Amplitude: $A = \frac{1}{2}(Maximum - Minimum)$
    - Period $= \frac{2\pi}{b}$, When looking at a graph, when have we finished one cycle/patten?
    - Frequency = $\frac{b}{2\pi}$, reciprocal of period. When looking at the graph, how many cycles/patterns in 2π
    - Vertical Shift (k): Where is the midline in relation to the x-axis
    - Writing an equation Generic From:
        - y= a sin(bx) +k

Unit 9: Statistics

Collecting Data:

- Surveys
- Observational Studies
- Experiments

Types of Sampling:

- Simple Random Sample: All in population have same probability of being selected.
- Stratified Random Sample: Population is divided into subgroups, then each subgroup has individuals that are randomly selected.
- Systematic Random Sample: Every nth person is chosen to participate.
- Convenience Sampling: The easiest group to obtain (the first 10 visitors)
- Cluster Sample: Population is divided into groups, then a random clusters are included in sample.

- Voluntary Response: Individuals choose to participate.

Types of Bias:

- Question Wording Bias: When the wording of a question affects the way someone responds.
- Undercoverage Bias: When the sample is not representative of the population
- Response Bias: When respondents lie or misrepresent themselves
- Nonresponse Bias: When an individual is chosen to participate, but refuses.
- Voluntary Response Bias: Those who choose to participate usually have extreme feelings in one direction.

Parameter vs. Statistic:

- Parameter: a value that represents a population.
- Statistic: a value based on a sample that is used to estimate a parameter.

Confidence Intervals:

- We cannot guarantee that a sample represents the population, so we use Margin of Error (MOE) to create an interval in which we feel confident the parameter will lie.
- Margin of Error: $\frac{1}{\sqrt{n}}$, where n is the sample size.

Simulations:

- A way to model random events so that simulated outcomes closely match real world outcomes.
    - Math → PRB → randInt()