Unit 2 Day 1: Introduction to Logarithms and Solving Log Equations

- Let's say that we wanted to solve an equation where the variable is in the exponent, such as $4^x = 12$
- We need a way to "undo" the base of 4.
- This inverse operation is called a logarithm

b = base
x = exponent

swoosh

Why? Helps us to work with really big & small numbers.

If $y = b^x$, then $\log_b y = x$

"x is the power, I raise b to to get y"

Ex. Rewrite each equation in its equivalent exponential form:

1. $\log_5 x = 2$ → $5^2 = x$
2. $2 = \log_b 25$ → $b^2 = 25$
3. $\log_3 7 = r$ → $3^r = 7$

Ex. Rewrite each equation in its equivalent logarithmic form:

1. $12^2 = x$ → $\log_{12} x = 2$
2. $b^3 = 27$ → $\log_b 27 = 3$
3. $4^y = 9$ → $\log_4 9 = y$

The Common Log - has a base of 10 ($\log_{10} y = x$ but we write $\log y = x$)

Change of Base!

Evaluating Logarithmic Expressions

Ex 1) Evaluate $\log_5 25$ → $\frac{\log(25)}{\log(5)} = 2$

Ex 2) $\log_4 64$ → $\frac{\log(64)}{\log(4)} = 3$

Ex 3) $\log_6 1296$ → $4 = \frac{\log(1296)}{\log(6)}$

Change of Base: $\frac{\log(\text{Big Font})}{\log(\text{small font})}$

Ex 4) $\log 100$ → $\frac{\log(100)}{\log(10)} = 2$

Ex 5) $\log_5 5$ → $\frac{\log(5)}{\log(5)} = 1$

Ex 6) $\log_3 1$ → $\frac{\log(1)}{\log(3)} = 0$

Solving Logarithmic Equations

Ex 1) $\log_3(6x+3) = 5$

$3^5 = 6x+3$

$243 = 6x+3$ -3 -3

$240 = 6x$

$\frac{240}{6} = \frac{6x}{6}$

$x = 40$

Ex 2) $\log_x 47 = 2$

$\sqrt{x^2} = \sqrt{47}$

$x = 6.86$

Ex 3) $\log_4(7x-5) = \log_4(5x+3)$

$7x - 5 = 5x + 3$

$+5 \quad +5$

$7x = 5x + 8$

$-5x \quad -5x$

$2x = 8$

$x = 4$

Ex 4) $\log_x 216 = 3$

$x^3 = 216$

$x = 6$

Ex 5) $\log_9(11x+14) = 3$

$9^3 = 11x + 14$

$x = 65$

Ex 6) $\log_8(-5x+4) = \log_8(13x-5)$

$-5x + 4 = 13x - 5$

$x = \frac{1}{2}$ or $.5$

$$y = b^x$$

* b stays the base for log too!
* x & y switch

$$\log_b y = x$$

$$\log_b y = x$$

* log cancels
* b to the power of x

$$y = b^x$$