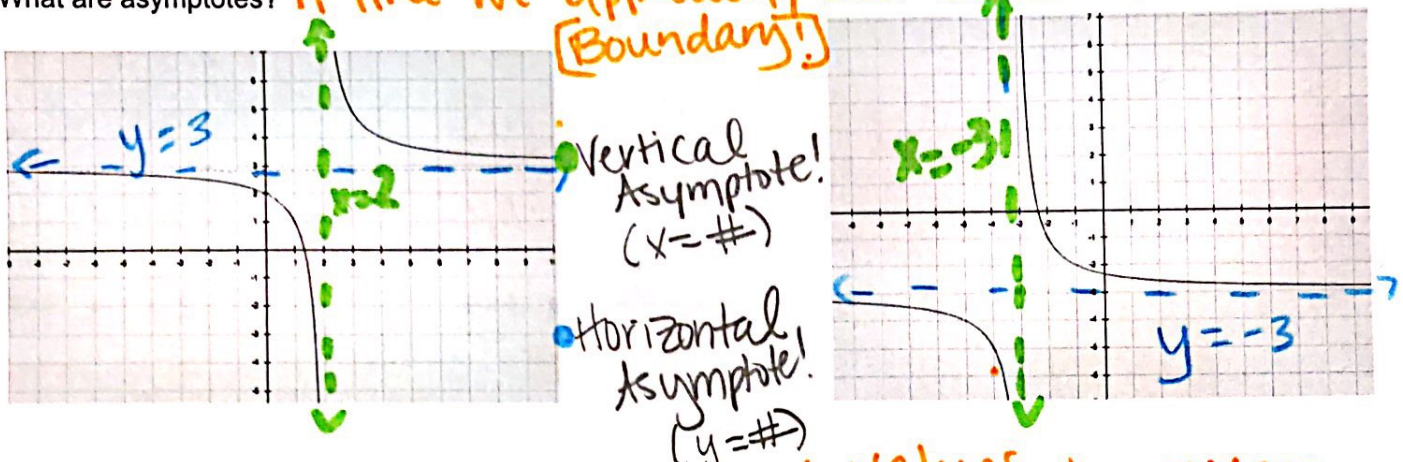


GUIDED NOTES: Asymptotes of Rational Functions

What are asymptotes? **A line we approach, but never cross!**
[Boundary!]



Vertical Asymptote!
 $(x = \#)$

Horizontal Asymptote!
 $(y = \#)$

DOMAIN: The domain of a function is the set of all possible **x-values; however**
restrictions (Vertical Asymptote, Hole) are values not included in the domain!

Vertical Asymptotes: (VA)

To find vertical asymptotes: **(after simplifying) set factors in denominator = 0!**

EX1. $y = \frac{x-3}{(x+2)(x-2)}$

$x+2=0$ $x-2=0$
 $-2 \quad -2$ $+2 \quad +2$

Domain:
 all real
 #s
 $x \neq -2, 2$

VA: $x = -2$ AND $x = 2$

EX2. $y = \frac{x-1}{x^2+5x+4} = \frac{x-1}{(x+4)(x+1)}$

$x+4=0$ $x+1=0$

Domain:
 All real
 #s
 $x \neq -4, -1$

VA: $x = -4$ AND $x = -1$

Holes:

What is the Vertical Asymptote of $y = \frac{x(x-4)}{x-4}$? Graph it, what do you notice? **same as $y=x$**

A **HOLE** in the graph is when $(x-a)$ is a **factor** in both the numerator and the denominator **(It cancels out!!!)**

EX3. $y = \frac{x^2-1}{x^2-2x-3} = \frac{(x-1)(x+1)}{(x-3)(x+1)} = \frac{x-1}{x-3}$

Hole: $x = -1$

VA: $x = 3$

Domain: all real #s
 $x \neq -1, 3$

EX4. $y = \frac{12x+24}{x^2+2x} = \frac{12(x+2)}{x(x+2)}$ Hole = $\frac{12}{x}$
 \downarrow
 $x=0$

Hole: $x = -2$

VA: $x = 0$

Domain: All real #s
 $x \neq -2, 0$

BOBO

Horizontal Asymptotes: (HA)

To find a horizontal asymptote, we focus on the degree \rightarrow highest exponent. of the numerator and the denominator!

BOBO

Big On Bottom

HA: $y = 0$

Examples:

EX5. $y = \frac{6x-5}{3x+9}$

BOTN

BOTN

Big On Top

HA: None!

EX6. $y = \frac{4x^6+3x^4-4x^2}{3x^1}$

BOTN

EATS DC

EATS DC

Exponents Are The Same

Divide Coefficients

EX7. $y = \frac{(x-3)(x+4)^2}{(x-5)^3(x+2)^2}$

3 # in front of x!
5

BOBO

EATS DC

Divide Coefficients!

HA: $y = 6/3 = 2$

$y = 2$

HA: none!

HA: $y = 0!$

EVERYTHING ALL TOGETHER:

Find the holes, vertical asymptotes, domain, and horizontal asymptotes for each rational function.

EX8. $y = \frac{5x^2+20x}{x^2+11x+28} = \frac{5x(x+4)}{(x+7)(x+4)} = \frac{5x}{x+7}$

Holes: $x = -4$

VA: $x = -7$

Domain: All real #s
 $x \neq -4, -7$

HA: EATS DC

$y = 5/1 = 5$

$y = 5$

EX9. $y = \frac{2x^2+10x+12}{x^3+3x^2+2x} = \frac{2(x+3)(x+2)}{x(x+2)(x+1)}$

Holes: $x = -2$

VA: $x = 0, -1$

Domain: All real #s

$x \neq -2, 0, -1$

HA: BOBO

$y = 0$