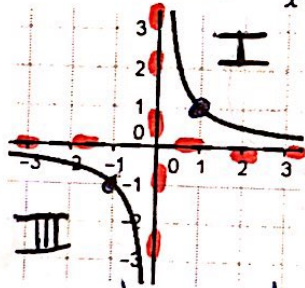


Graphing Rational Functions

Graphing Rational Functions Using Transformations

Parent graph: $y = \frac{1}{x}$



Shape called a hyperbola
Horizontal asymptote at $y = 0$
Vertical asymptote at $x = 0$
Domain: $x \neq 0$ Range: $y \neq 0$
Passes through $(1, 1)$ and $(-1, -1)$

Range: $y \neq 0$

* Domain & Range are restricted by asymptotes

B/c of asymptotes our function is discontinuous

↑
2 parts /
made up of
2 sections

Graphing Rational Functions

Example 1 Describing with transformations

$y = \frac{a}{x-h} + k$
 $a \geq 1$ the function decrease from left to right
 $a < 0$ reflects the graph in the x-axis so the function increase from left to right (II, IV)
 $a/1$ is the slope from the intersection of the asymptotes to the points one unit left and right
 $|a| > 1$ stretch $|a| < 1$ shrink
 $+k$ shifts the graph UP
 $-k$ shifts the graph DOWN
 Horizontal asymptote $y=k$. The range is $y \neq k$.
 $(x-h)$ shifts the graph RIGHT
 $(x+h)$ shifts the graph LEFT
 Vertical asymptote $x=h$. The domain is $x \neq h$.

*** Domain & Range are restricted by asymptotes**

Describe each graph as compared to the parent graph $y=1/x$.

A) $y = \frac{1}{x+5} - 7$ *decrease*
 Left 5
 Down 7
 Domain $x \neq -5$
 Range: $y \neq -7$

B) $y = \frac{-1}{x} + 4$ *increasing*
 UP 4
 Reflection in x-axis
 Domain $x \neq 0$
 Range $y \neq 4$

C) $y = \frac{6}{x-6}$
 Right 6
 decreases
 stretches by factor of 6
 Domain $x \neq 6$
 Range: $y \neq 0$

B/c of asymptotes our function is discontinuous
 ↑
 2 parts / made up of 2 sections

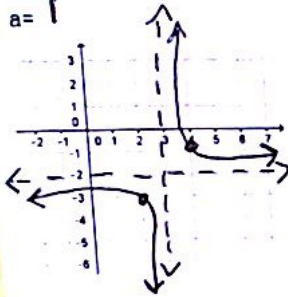
Graphing Rational Functions

Example 2 Graph using transformations

Graph the asymptotes, use $a/1$ to find the two guide points, and sketch the graph.

A) $y = \frac{1}{x-3} - 2$

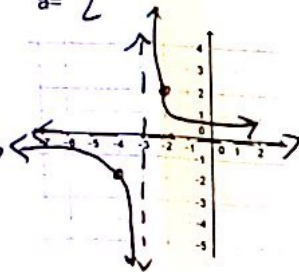
HA: $y = -2$ VA: $x = 3$
 $a = 1$



Domain: $x \neq 3$
 Range: $y \neq -2$

B) $y = \frac{2}{x+3}$

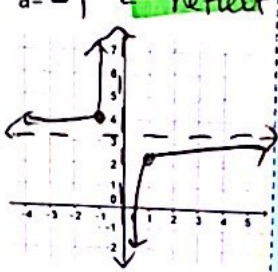
HA: $y = 0$ VA: $x = -3$
 $a = 2$



Domain: $x \neq -3$
 Range: $y \neq 0$

C) $y = \frac{-1}{x} + 3$

HA: $y = 3$ VA: $x = 0$
 $a = -1$ ← reflect



Domain: $x \neq 0$
 Range: $y \neq 3$

Domain & Range are restricted by asymptotes

of asymptotes our function is discontinuous

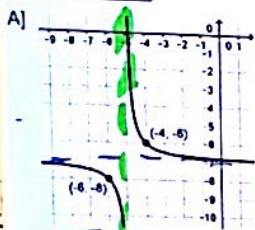
↑
 2 parts / made up of 2 sections

Graphing Rational Functions

Example 3 Write the equation of the rational function

$$y = \frac{a}{x-h} + k$$

The vertical asymptote is exactly half way between the x-coordinates of the guide points.
The horizontal asymptote is exactly half way between the y-coordinates of the guide points.
Identify the values of h, k, and a from the graph. Then use them to write the equation.

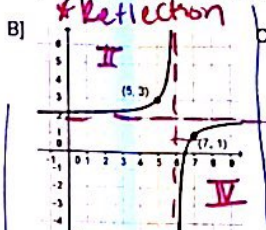


$$h = -5$$

$$k = -7$$

$$a = 1$$

$$y = \frac{1}{x+5} - 7$$

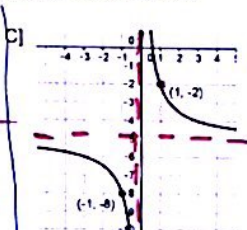


$$h = 6$$

$$k = 2$$

$$a = -1$$

$$y = \frac{-1}{x-6} + 2$$



$$h = 0$$

$$k = -5$$

$$a = 3$$

$$y = \frac{3}{x} - 5$$

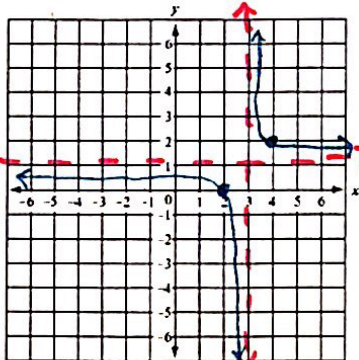
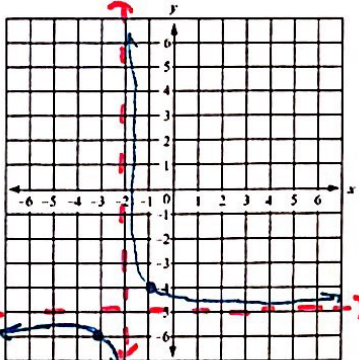
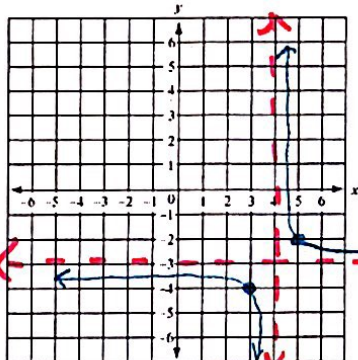
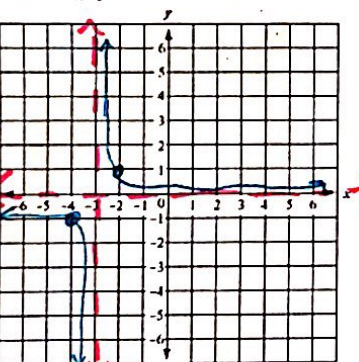
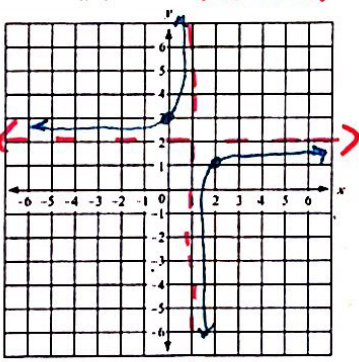
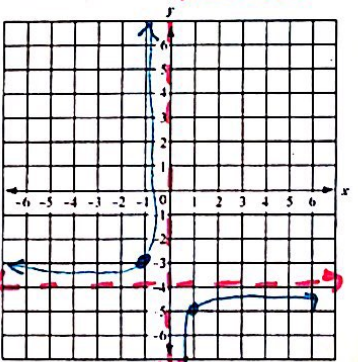
Domain & Range are restricted by asymptotes

B/c of asymptotes our function is discontinuous

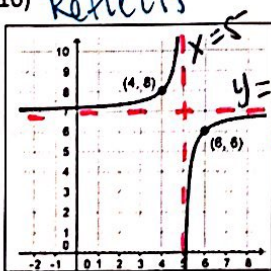
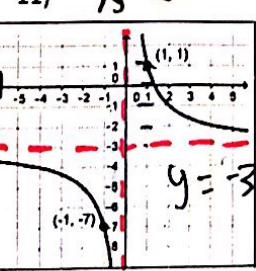
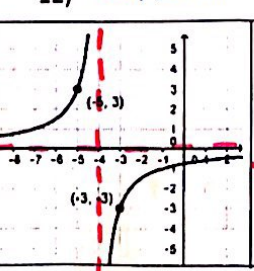
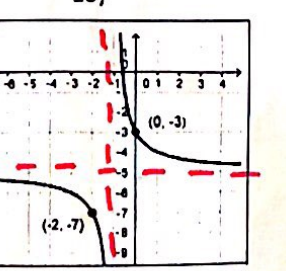
↑
2 parts / made up of 2 sections

Practice: Graphing Rational Functions

Graph each rational function.

<p>1) $y = \frac{1}{x-3} + 1$</p> 	<p>2) $y = \frac{1}{x+2} - 5$</p> 	<p>3) $y = \frac{1}{x-4} - 3$</p> 
<p>Domain: $x \neq 3$</p> <p>Range: $y \neq 1$</p> <p>H.A.: $y = 1$</p> <p>V.A.: $x = 3$</p>	<p>Domain: $x \neq -2$</p> <p>Range: $y \neq -5$</p> <p>H.A.: $y = -5$</p> <p>V.A.: $x = -2$</p>	<p>Domain: $x \neq 4$</p> <p>Range: $y \neq -3$</p> <p>H.A.: $y = -3$</p> <p>V.A.: $x = 4$</p>
<p>4) $y = \frac{1}{x+3}$</p> 	<p>5) $y = \frac{-1}{x-1} + 2$ Reflect</p> 	<p>6) $y = \frac{-1}{x} - 4$ Reflect</p> 
<p>Domain: $x \neq -3$</p> <p>Range: $y \neq 0$</p> <p>H.A.: $y = 0$</p> <p>V.A.: $x = -3$</p>	<p>Domain: $x \neq 1$</p> <p>Range: $y \neq 2$</p> <p>H.A.: $y = 2$</p> <p>V.A.: $x = 1$</p>	<p>Domain: $x \neq 0$</p> <p>Range: $y \neq -4$</p> <p>H.A.: $y = -4$</p> <p>V.A.: $x = 0$</p>

Write an equation for each rational function.

<p>10) Reflects</p> 	<p>11) $x=0$</p> 	<p>12) Reflects</p> 	<p>13)</p> 
<p>$y = \frac{-1}{x-5} + 7$</p>	<p>$y = \frac{4}{x} - 3$</p>	<p>$y = \frac{-3}{x+4}$</p>	<p>$y = \frac{2}{x+1} - 5$</p>