

# 6.2 Equations of Circles

*h = x-value of center  
k = y-value of center*

SWBAT graph circles on the coordinate plane and write the equations of circles in standard form.

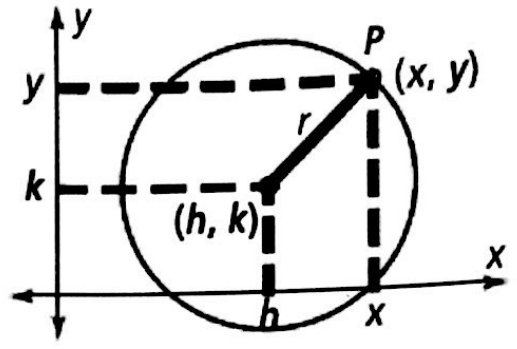
## Standard Form of Circles

$$(x-h)^2 + (y-k)^2 = r^2$$

Center:  
 $(h, k)$

Radius:  
 $r$

Point on the circle:  
 $(x, y)$



*Opposite value!*

**Example 1:** Write the equation of a circle with the given information.

- a) Center  $(0,0)$ , Radius  $=10$   
 $h = 0$     $k = 0$     $r = 10$

$$x^2 + y^2 = 100$$

- b) Center  $(2, 3)$ , Diameter  $=12 \Rightarrow r = 6$   
 $h = 2$     $k = 3$     $r = 6$   
 $(x-2)^2 + (y-3)^2 = 6^2$   
 $(x-2)^2 + (y-3)^2 = 36$

**Example 2:** Determine the center and radius of a circle the given equation.

- a)  $x^2 + y^2 = \frac{9}{4}$   
 C:  $(0,0)$   
 R:  $1.5$  or  $3/2$

- b)  $(x+3)^2 + (y-5)^2 = 81$   
 $\sqrt{81} = 9$   
 C:  $(-3, 5)$   
 R:  $9$

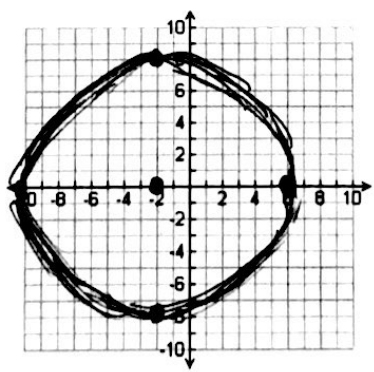
- c)  $(x+4)^2 + (y+6)^2 = 1$   
 C:  $(-4, -6)$   
 R:  $1$

**Example 3:** Use the center and the radius to graph each circle.

a)  $(x+2)^2 + y^2 = 64$

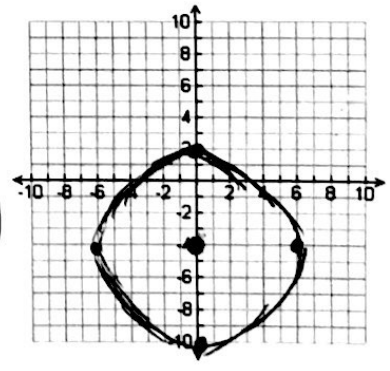
Center:  $(-2, 0)$   
 Radius:  $8$

$\sqrt{64} = 8$



b)  $x^2 + (y+4)^2 = 36$

Center:  $(0, -4)$   
 Radius:  $6$



Do NOT make straight lines!!  
 Create curves/arches

Writing an Equation with a Pass-Thru Point	
Step 1:	Substitute the center (h, k) into the equation
Step 2:	Substitute the "pass through point (x, y)" into the equation for x and y.
Step 3:	Simplify and solve for r <sup>2</sup> .
Step 4:	Substitute r <sup>2</sup> back into the equation from Step 1.

$$(x-h)^2 + (y-k)^2 = r^2$$

①  $(x-2)^2 + (y-5)^2 = r^2$   
 ②  $(5-2)^2 + (-1-5)^2 = r^2$   
 ③  $9 + 36 = r^2$   
 $45 = r^2$

**Example 4:** Write the equation of a circle with a given center (2, 5) that passes through the point (5, -1). \*We do not change the sign of the pass through pt.  
 What's the radius?  
 $\sqrt{45} = 6.70$

④  $(x-2)^2 + (y-5)^2 = 45$

Writing an Equation with Two Points on the Circle	Midpoint Formula
Find the midpoint (radius) between the two endpoints, and then follow steps 1-4.	$(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$

**Example 5:** Write the equation of a circle with endpoints of diameter at (-6, 5) and (4, -3).

Center:  $(\frac{-6+4}{2}, \frac{5+(-3)}{2}) = (-1, 1)$   
 $(x+1)^2 + (y-1)^2 = r^2$   
 $(-6+1)^2 + (5-1)^2 = r^2$   
 $(-5)^2 + (4)^2 = r^2$   
 $25 + 16 = r^2$   
 $r^2 = 41$   
 $(x+1)^2 + (y-1)^2 = 41$

Writing the Equation of a Circle in Standard Form	
Step 1:	Group x's and group y's together.
Step 2:	Move any constants to the right side of the equation.
Step 3:	Use complete the square to make a perfect square trinomial for the x's and then again for the y's. *Remember, whatever you do to one side of the equation, you must do to the other!
Step 4:	Simplify factors into standard form of a circle!

**Example 5:** Write the equation of a circle in standard form. Then, state the center and the radius.

a)  $x^2 + y^2 + 4x - 8y + 16 = 0$   
 $(x^2 + 4x) + (y^2 - 8y) = -16$   
 $(x+2)^2 + (y-4)^2 = 4$

Center: (-2, 4)  
 Radius: 2

b)  $x^2 + y^2 + 6x - 4y = 0$   
 $(x^2 + 6x) + (y^2 - 4y) = 0$   
 $(x+3)^2 + (y-2)^2 = 13$

Center: (-3, 2)  
 Radius:  $\sqrt{13}$

c)  $x^2 + y^2 - 6x - 2y + 4 = 0$   
 $(x-3)^2 + (y-1)^2 = 6$

Center: (3, 1)  
 Radius:  $\sqrt{6}$

d)  $x^2 + y^2 + 8x - 10y - 4 = 0$   
 $(x+4)^2 + (y-5)^2 = 45$

Center: (-4, 5)  
 Radius:  $\sqrt{45}$