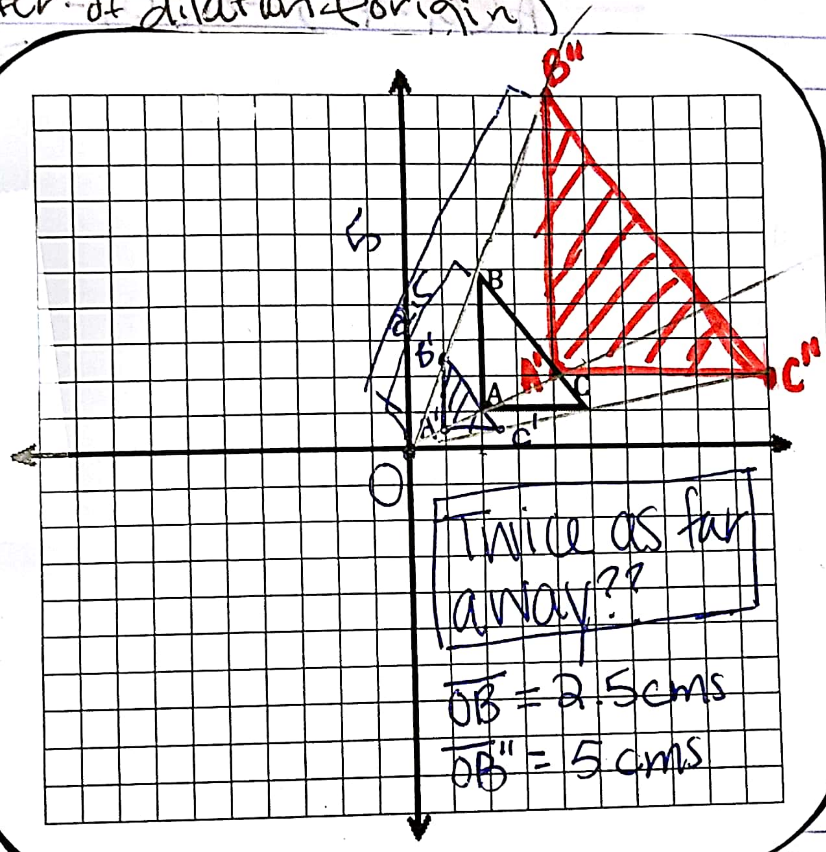
Dilations

A transformation that enlarges or shrinks a figure from a given point called the center of dilation (origin)



Twice as far away??

$\overline{OB} = 2.5$ cms

$\overline{OB''} = 5$ cms

* Dilations create similar figures! NOT an isometry

Quadratic Functions

Dilate ΔABC by a scale factor of $\frac{1}{2}$ with the origin as the center of dilation

$\Delta ABC \rightarrow \Delta A'B'C'$
$A(2,1) \rightarrow A'(1, \frac{1}{2})$
$B(2,5) \rightarrow B'(1, 2.5)$
$C(5,1) \rightarrow C'(2.5, \frac{1}{2})$

$(x,y) \rightarrow (\frac{1}{2}x, \frac{1}{2}y)$

* Half the size! & half ~~the~~ as far away!

Scale Factors

<1 shrinks, smaller, reduction

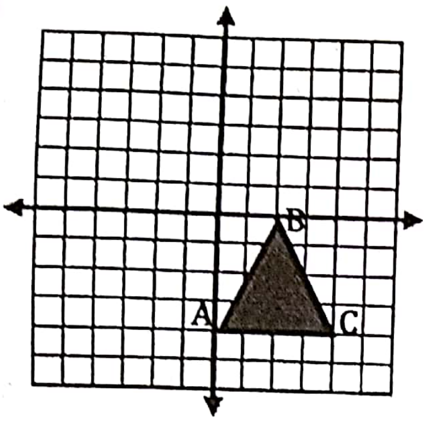
>1 bigger, enlarges, stretches.

Dilate ΔABC by a scale factor of 2 with the origin as the center of dilation

$\Delta ABC \rightarrow \Delta A''B''C''$
$A(2,1) \rightarrow A''(4,2)$
$B(2,5) \rightarrow B''(4,10)$
$C(5,1) \rightarrow C''(10,2)$

$(x,y) \rightarrow (2x, 2y)$

* TWICE the size! Twice as far away



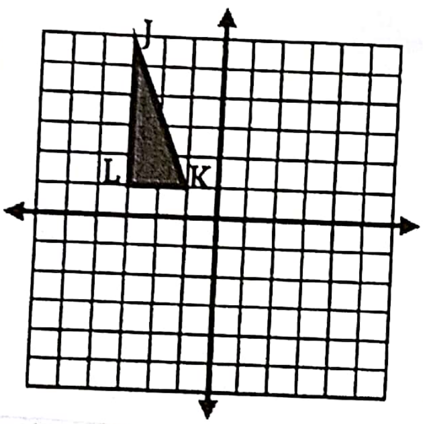
Dilate $\triangle ABC$ by a scale factor of 1/4.

$(x, y) \rightarrow (\frac{1}{4}x, \frac{1}{4}y)$

$A(0, -4) \rightarrow A'(0, -1)$

$B(2, 0) \rightarrow B'(\frac{1}{2}, 0)$

$C(4, -4) \rightarrow C'(1, -1)$



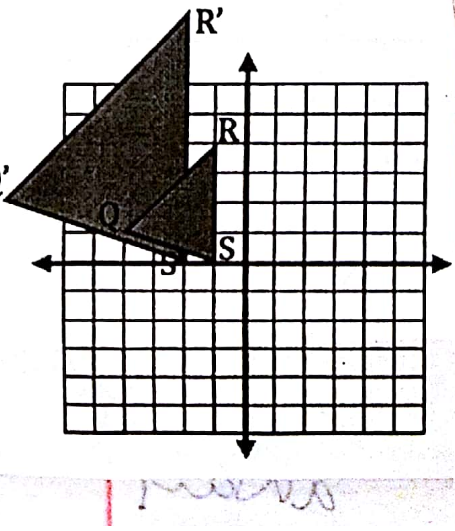
Dilate $\triangle JKL$ by a scale factor of 2

$(x, y) \rightarrow (2x, 2y)$

$J(-3, 6) \rightarrow J'(-6, 12)$

$K(-1, 1) \rightarrow K'(-2, 2)$

$L(-3, 1) \rightarrow L'(-6, 2)$



What was the scale factor?

$S(-1, 0) \rightarrow S'(-2, 0)$

$(x, y) \rightarrow (2x, 2y)$

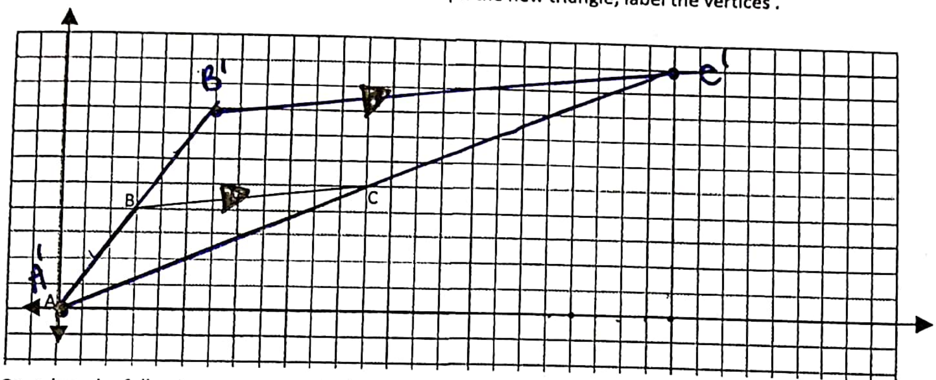
Scale factor: 2

Dilations (Again!)

$(x, y) \to (2x, 2y)$ A(0,0) → A'(0,0)
B(3,4) → B'(6,8)
C(12,5) → C'(24,10)

Properties of Dilation Investigation

Dilate about the origin with a magnitude of 2. Graph the new triangle; label the vertices.



Complete the following using your dilation.

1. Compare the angles of △ABC and △A'B'C'. What do you notice? the angles look same!

2. Using the distance formula, calculate the lengths of What do you notice? (Hint: you may want to round to the tenths place to make the comparison easier)

$\overline{AB} = \sqrt{9+16} = 5$ $\overline{BC} = \sqrt{82} \approx 9.06$ $\overline{AC} = \sqrt{169} = 13$

$\overline{A'B'}: \sqrt{36+64} = 10$ $\overline{B'C'} = \sqrt{328} \approx 18.11$ $\overline{A'C'} = \sqrt{676} = 26$

3. Dilations create **similar figures.** Based on your observations from 1 and 2, what can we conlude about similar figures? → angles are congruent
→ side lengths are proportional

4. What do you notice about the placement of $\overline{AC}$ and $\overline{A'C'}$ on the coordinate plane?
Note that *A* and *A'* lie on the origin. What conclusion can you make about the segments of an image when the corresponding segments of the preimage pass through the center of dilation?
- $\overline{AC}$ & $\overline{A'C'}$ are a part of the same line.
- $\overline{AB}$ & $\overline{A'B'}$ are part of the same line

5. Using the slope formula, calculate the slopes of $\overline{BC}$ and $\overline{B'C'}$. What do you notice about the slopes? What does that tell you about the relationship of the lines to one another? What conclusion can you make about the segments of an image when the corresponding segments of the preimage do not pass through the center of dilation?

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$ $\overline{BC} = \dfrac{5-4}{12-3} = \boxed{\dfrac{1}{9}} = .111$

Checkpoint:

$\overline{B'C'} = \dfrac{10-8}{24-6} = \dfrac{2}{18} = \boxed{\dfrac{1}{9}}$

$\boxed{\text{*parallel*}}$

$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

13

Checkpoint:

When a line segment passes through the center of dilation, the line segment and its image lie on the same line.

When a line segment does not pass through the center of dilation, the line segment and its image are parallel.

Dilations create figures that are always similar to one another.

Two figures are similar (~) if they have the same ∠ measures but not necessarily the same side lengths

same shape!

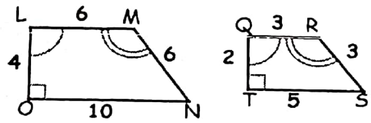
The scale factor is the ratio of the lengths of the corresponding sides.

(a.k.a. the magnitude)

Two figures are congruent (≅) if they are similar and have the same side lengths

Two polygons are similar if:

1) Corresponding ∠s are ≅ AND 2) Corresponding sides are proportional



$\frac{6}{3} = 2$ $\frac{6}{3} = 2$

□LMNO □QRST

2 : 1