Unit 6 – Day 5: Conditional Probability

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Conditional Probability: -

A probability where a certain prerequisite has already been met.

The conditional probability of A given B is expressed as $P(A|B)$

The formula is: $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$ ← intersection ← given

Examples of Conditional Probability:

1. You are playing a game of cards where the winner is determined by drawing two cards of the same suit. What is the probability of drawing clubs on the second draw *if the first card drawn is a club?*

$P(\text{2nd club} \mid \text{1st club}) = \dfrac{P(\text{2nd club} \cap \text{1st club})}{P(\text{1st club})} = \dfrac{\frac{13}{52} \cdot \frac{12}{51}}{\frac{13}{52}} = \dfrac{12}{51}$

$\boxed{\dfrac{12}{51}}$

8 total

2. A bag contains 6 blue marbles and 2 brown marbles. One marble is randomly drawn and discarded. Then a second marble is drawn. Find the probability that the second marble is brown given that *the first marble drawn was blue.*

$P(\text{2nd brown} \mid \text{1st blue}) = \dfrac{\frac{6}{8} \cdot \frac{2}{7}}{\frac{6}{8}} = \boxed{\dfrac{2}{7}} = 28.57\%$

$= .2857$

3. In Mr. Jonas' homeroom, 70% of the students have brown hair, 25% have brown eyes, and 5% have both brown hair and brown eyes. A student is excused early to go to a doctor's appointment. *If the student has brown hair,* what is the probability that the student also has brown eyes?

$P(\text{brown eyes} \mid \text{brown hair}) = \dfrac{P(\text{brown eyes} \cap \text{brown hair})}{P(\text{brown hair})}$ ← given

$= \dfrac{.05}{.7} = 0.714 = 7.14\%$

Conditional Probability

Using Two-Way Frequency Tables to Compute Conditional Probabilities

1. Suppose we survey all the students at school and ask them how they get to school and also what grade they are in. The chart below gives the results. Complete the two-way frequency table:

	Bus	Walk	Car	Other	Total
9th or 10th	106	30	70	4	210
11th or 12th	41	58	184	7	290
Total	147	88	254	11	500

Suppose we randomly select one student.

a. What is the probability that the student walked to school? $\frac{88}{500} = .176 = 17.6\%$

b. P(9th or 10th grader) $\frac{210}{500} = .42 = 42\%$

c. P(rode the bus OR 11th or 12th grader) $= P(\text{bus}) + P(11/12) - P(\text{bus} \cap 11/12)$

$$\frac{147}{500} + \frac{290}{500} - \frac{41}{500} = \frac{396}{500} = 79.2\%$$

d. What is the probability that a student is in 11th or 12th grade *given that they rode in a car to school?*

$$P(11/12 \mid \text{car}) = \left(\frac{184}{500}\right) / \left(\frac{254}{500}\right) = .724 = 72.4\%$$

e. What is P(Walk|9th or 10th grade)?

$$30/210 = 14.2\%$$

2. The manager of an ice cream shop is curious as to which customers are buying certain flavors of ice cream. He decides to track whether the customer is an adult or a child and whether they order vanilla ice cream or chocolate ice cream. He finds that of his 224 customers in one week that 146 ordered chocolate. He also finds that 52 of his 93 adult customers ordered vanilla. Build a two-way frequency table that tracks the type of customer and type of ice cream.

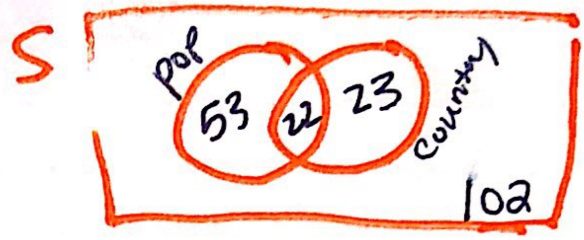
	Vanilla	Chocolate	Total
Adult	52	41	93
Child	26	105	131
Total	78	146	224

a. Find P(vanilla|adult) $\frac{52}{93} = 55.9\%$

b. Find P(child|chocolate)

$$\frac{105}{146} = 71.9\%$$

Conditional Probability (cont)



3. A survey asked students which types of music they listen to? Out of 200 students, 75 indicated pop music and 45 indicated country music with 22 of these students indicating they listened to both. Use a Venn diagram to find the probability that a randomly selected student listens to pop music given that they listen country music.

$$P(\text{pop} \mid \text{country}) = \frac{\frac{22}{200}}{\frac{45}{200}} = 48.9\%$$

Using Conditional Probability to Determine if Events are Independent

If two events are statistically independent of each other, then:

$P(A|B) = P(A)$, if so, then indep.

Let's revisit some previous examples and decide if the events are independent.

1. You are playing a game of cards where the winner is determined by drawing two cards of the same suit without replacement. What is the probability of drawing clubs on the second draw if the first card drawn is a club?

- Are the two events independent?
- Let drawing the first club be event A and drawing the second club be event B.

$P(\text{2nd club} \mid \text{1st club}) = 12/51$

$P(\text{club}) = 13/52$

$\frac{12}{51} \neq \frac{13}{52}$

dep. not indep

2. You are playing a game of cards where the winner is determined by drawing tow cards of the same suit. Each player draws a card, looks at it, then replaces the card randomly in the deck. Then they draw a second card. What is the probability of drawing clubs on the second draw if the first card drawn is a club? Are the two events independent?

Independent. $\frac{13}{52}$

3. In Mr. Jonas' homeroom, 70% of the students have brown hair, 25% have brown eyes, and 5% have both brown hair and brown eyes. A student is excused early to go to a doctor's appointment. If the student has brown hair, what is the probability that the student also has brown eyes?

- Are event A, having brown hair, and event B, having brown eyes, independent?

$P(\text{brown eyes} \mid \text{brown hair}) = .0714$

$P(\text{brown eyes}) = .25$

not independent