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Completing the □

COMPLETING THE SQUARE! (PUTTING QUADRATICS INTO VERTEX FORM)

Let's look at $y = x^2 + 6x + 9$. This is called a perfect square **That means that I can easily write this one in factored and vertex form.**

Ex. 1

a. Factor $y = x^2 + 6x + 9$ to put it into intercept form: $y = (x+3)(x+3)$

$+b$: $3+3$ $\quad$ $\times 9$: $3 \cdot 3$

b. How can I write that intercept form equation in vertex form?

$y = (x+3)^2$

Because I had a perfect square trinomial, I could put $y = x^2 + 6x + 9$ into vertex form.

A perfect square trinomial is one that can be written as $(x \pm h)^2$

Ex. 2

a. Is $y = x^2 + 10x + 25$ a perfect square? Why or why not? yes! $5+5=10$ $5 \cdot 5 = 25$

b. Write $y = x^2 + 10x + 25$ in intercept form. $y = (x+5)(x+5)$

c. Write $y = x^2 + 10x + 25$ in vertex form: $y = (x+5)^2$

What if I am looking at a trinomial that is not a perfect square? Can I put that into vertex form?

Ex 3.

a. Is $y = x^2 + 6x + 7$ a perfect square? Why or why not? NO!! There are no two numbers that multiply to 7 and add to 6

b. We can make part of it into a perfect square. Let's COMPLETE THE □

$y = (x^2 + 6x + \underline{9}) + 7 - \underline{9}$

$\frac{b}{2} = 3 \rightarrow 3^2 = 9$

$y = (x+3)^2 + 7 - 9$

* You must also subtract to maintain equality.

c. Now let's write the perfect square trinomial in binomial notation and simplify!

Vertex form: $(x+3)^2 - 2$

Vertex: $(-3, -2)$

$y = x^2 + 6x + 7$

YOU TRY!

Put the following standard form equations into vertex form.

1. $y = x^2 + 10x + 24$

$\frac{10}{2} = 5$

$y = (x^2 + 10x + 25) + 24 - 25$

$y = (x+5)^2 - 1$

2. $y = x^2 + 8x + 19$

$\frac{8}{2} = 4$

$y = (x^2 + 8x + 16) + 19 - 16$

$y = (x+4)^2 + 3$

3. $y = x^2 - 6x + 10$

$\frac{-6}{2} = -3 \rightarrow (-3)^2 = 9$

$y = (x^2 - 6x + 9) + 10 - 9$

$y = (x-3)^2 + 1$

4. $y = x^2 - 12x - 6$

$\frac{-12}{2} = -6 \rightarrow (-6)^2 = 36$

$y = (x^2 - 12x + 36) - 6 - 36$

$y = (x-6)^2 - 42$

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COMPLETING THE SQUARE WHEN a IS NOT 1!

1. $y = 3x^2 + 9x + 6$

$3[x^2+3x+2]$ $\quad \frac{3}{2} = 1.5 \rightarrow 1.5^2 = 2.25$

$3[(x^2+3x+2.25)+2-2.25]$

$3[(x+1.5)^2-.25]$

$y = 3(x+1.5)^2 - .75$

2. $y = 2x^2 + 6x + 5$

$\frac{3}{2} = 1.5 \rightarrow 1.5^2 = 2.25$

$2[x^2+3x+2.5]$

$2[(x^2+3x+2.25)+2.5-2.25]$

$2[(x+1.5)^2+.25]$

$y = 2(x+1.5)^2 + .5$

3. $y = -x^2 + 5x - 4$

$-1[x^2-5x+4]$ $\quad \frac{-5}{2} = -2.5^2 = 6.25$

$-1[(x^2-5x+6.25)+4-6.25]$

$-1[(x-2.5)^2+2.25]$

$y = -(x-2.5)^2 + 2.25$

4. $y = x^2 - 7x + 4$

$-7/2 = (-3.5)^2 = 12.25$

$(x^2-7x+12.25)+4-12.25$

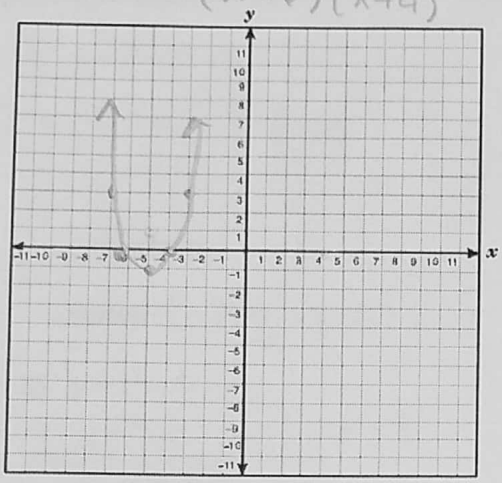
$y = (x-3.5)^2 - 8.25$

Sketch a graph by plotting the vertex, plotting the roots, and connecting them to make a parabola.

1) $y = x^2 + 10x + 24$

Vertex Form: $y = (x+5)^2 - 1$

Factored Form: $(x+6)(x+4)$

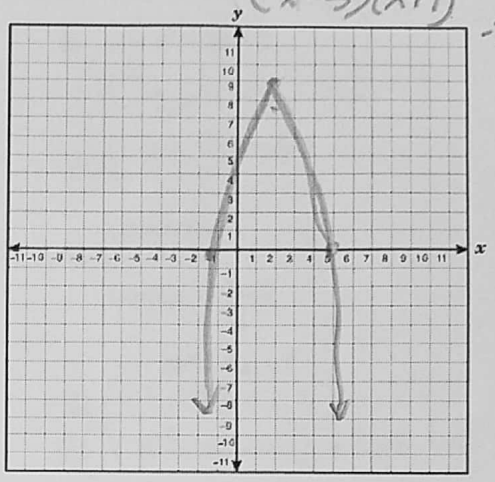


2) $y = -(x-2)^2 + 9$

Standard Form: $y = -x^2 + 4x + 5$

Factored Form: $-(x-5)(x+1)$

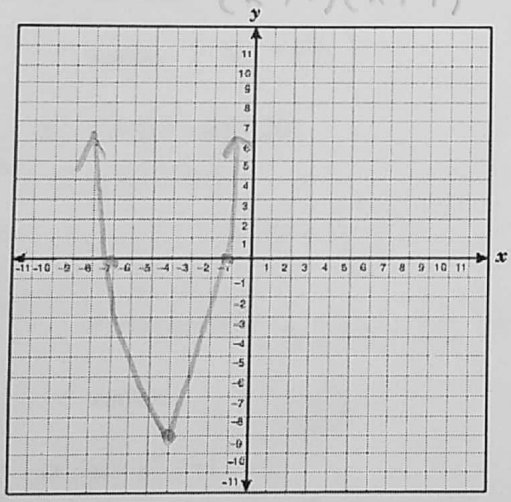
x	-2
x^2	-2x
-2x	+4



3) Standard Form: $y = x^2 + 8x + 7$

Vertex Form: $y = (x+4)^2 - 9$

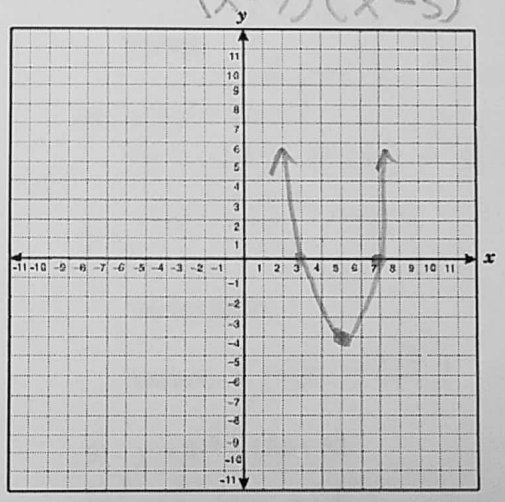
Factored Form: $(x+1)(x+7)$



4) Standard Form: $y = x^2 - 10x + 21$

Vertex Form: $y = (x-5)^2 - 4$

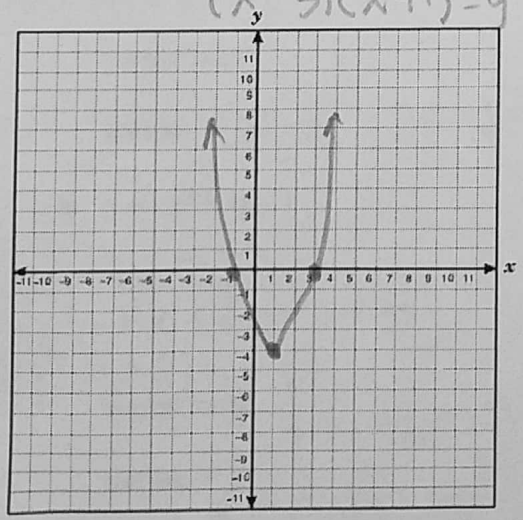
Factored Form: $(x-7)(x-3)$



5) Standard Form: $y = x^2 - 2x - 3$

Vertex Form: $y = (x-1)^2 - 4$

Factored Form: $(x-3)(x+1) = y$

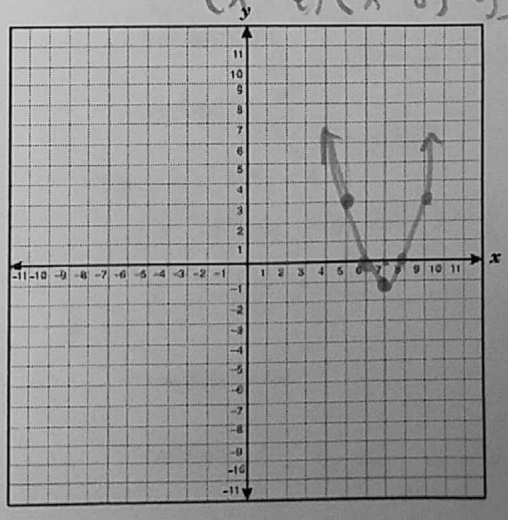


6) $y = (x-7)^2 - 1$

Standard Form: $x^2 - 14x + 48 = y$

Factored Form: $(x-6)(x-8) = y$

x	-7
x^2	-7x
-7x	49



Name: _____

Completing the Square

Rewrite each quadratic equation in vertex form by completing the square. Then state the vertex.

1) $y = x^2 + 7x + 4$

$Y = (x+3.5)^2 - 8.25$

Vertex form:

Vertex: $(-3.5, -8.25)$

2) $y = x^2 - 6x - 4$

$Y = (x^2 - 6x + 9) - 4 - 9$

$y = (x-3)^2 - 13$

Vertex form:

Vertex: $(3, -13)$

3) $y = 2x^2 - 8x + 20$

$2(x^2 - 4x + 10)$

$2[(x^2 - 4x + 4) + 10 - 4]$

$2[(x-2)^2 + 6]$

Vertex form: $y = 2(x-2)^2 + 12$

Vertex: $(2, 12)$

4) $y = x^2 - 11x + 5$

$Y = (x^2 - 11x + 30.5) + 5 - 30.25$

$y = (x - 5.5)^2 - 25.25$

Vertex form:

Vertex: $(5.5, -25.25)$

5) $y = -x^2 + 4x - 1$

$-(x^2 - 4x + 1)$

$-[(x^2 - 4x + 4) + 1 - 4]$

$-[(x-2)^2 - 3]$

Vertex form: $y = -(x-2)^2 + 3$

Vertex: $(2, 3)$

6) $y = x^2 + 2x + 2$

$(x^2 + 2x + 1) + 2 - 1$

$y = (x+1)^2 + 1$

Vertex form:

Vertex: $(-1, 1)$

7) $y = 2x^2 - 8x + 7$

$2(x^2 - 4x + 3.5)$

$2[(x^2 - 4x + 4) + 3.5 - 4]$

$2[(x-2)^2 - .5]$

Vertex form: $y = 2(x-2)^2 - 1$

Vertex: $(2, -1)$

8) $y = x^2 + 12x + 7$

$(x^2 + 12x + 36) + 7 - 36$

$y = (x+6)^2 - 29$

Vertex form:

Vertex: $(-6, -29)$

9) $y = 4x^2 - 16x + 52$

$4(x^2 - 4x + 13)$

$4[(x^2 - 4x + 4) + 13 - 4$

$4[(x-2)^2 + 9]$

Vertex form: $y = 4(x-2)^2 + 36$

Vertex: $(2, 36)$

10) $y = x^2 + 5x - 4$

$(x^2 + 5x + 6.25) - 4 - 6.25$

$y = (x + 2.5)^2 - 10.25$

Vertex form:

Vertex: $(-2.5, -10.25)$